3. domácí list

1 a) $$\lim_{n\to\infty}\left(\cos\frac{a}{n}\right)^{n^2} = \lim_{n\to\infty} e^{n^2\ln\left(\cos\frac{a}{n}\right)} \underset{H.V.}{=} \lim_{x\to\infty} e^{x^2\ln\left(\cos\frac{a}{x}\right)} \underset{\substack{VLSF\\ +\text{spoj. exp}}}{=} e^{-\frac{a^2}{2}}$$

$$\lim_{x\to+\infty} x^2\ln\left(\cos\frac{a}{x}\right) \underset{\substack{VLSF\\ \frac{1}{x}=t}}{=} \lim_{t\to0+}\frac{\overset{\to 1}{\ln(\cos at)}}{t^2} = \lim_{t\to0}\underbrace{\frac{\ln(\cos at)}{\cos at-1}}_{\to 1}\cdot\frac{\cos at-1}{t^2} = -\frac{a^2}{2}$$

$$\lim_{t\to0}\frac{\ln(\cos at)}{\cos at-1} \underset{\substack{VLSF\\ y=\cos at}}{=} \lim_{y\to1}\frac{\ln(y)}{y-1} = 1$$

$$\lim_{t\to0}\frac{\cos at-1}{t^2} = \lim_{t\to0}\frac{\cos^2 at-1}{t^2(\cos at+1)} = \lim_{t\to0}\underbrace{\frac{-(\sin at)^2}{a^2t^2}}_{\substack{\to -1\\ (VLSF+\text{tabulky})}}\cdot\underbrace{\frac{a^2}{(\cos at+1)}}_{\to\frac{a^2}{2}}$$

$$= -\frac{a^2}{2}$$

také l'H.

b) $$\lim_{n\to\infty} n\cdot\ln\left(1-\operatorname{arctg}\frac{3}{n}\right) \underset{H.V.}{=} \lim_{x\to\infty}\ln\left(1-\operatorname{arctg}\frac{3}{x}\right)\cdot x \quad \text{„}0\cdot\infty\text{"} = e^{-3}$$

$$\underset{\substack{VLSF\\ \frac{1}{x}=t}}{=} \lim_{t\to0+}\frac{\ln(1-\operatorname{arctg}3t)}{t} = \lim_{t\to0}\underbrace{\frac{\ln(1-\operatorname{arctg}3t)}{-\operatorname{arctg}3t}}_{\to 1}\cdot\frac{(-\operatorname{arctg}3t)}{t} =$$

$$= \lim_{t\to0}\underbrace{\frac{\ln(1-\operatorname{arctg}3t)}{-\operatorname{arctg}3t}}_{\to 1}\cdot\underbrace{\frac{+(\operatorname{arctg}3t)}{3t}}_{\to 1}\cdot(-3) = -3$$

2. b) ? lze spojitě dodef fci v bodě $a=0$?

(i) $\underline{f(x) = \operatorname{arctg}\frac{1}{x^2}}$, $x \neq 0$; $\lim\limits_{x\to 0} \operatorname{arctg}\frac{1}{x^2} \underset{VLSF}{=} \lim\limits_{t\to +\infty} \operatorname{arctg} t = \frac{\pi}{2}$,

lze tedy dodef. spojitě $f(0) = \frac{\pi}{2}$

(ii) $\underline{f(x) = \frac{1-\cos x}{x^3}}$; $\lim\limits_{x\to 0\pm} f(x) = \lim\limits_{x\to 0\pm} \underbrace{\frac{1-\cos x}{x^2}}_{\to \frac{1}{2}} \cdot \underbrace{\frac{1}{x}}_{\to \pm\infty} = \pm\infty$

tedy f nelze spojitě dodef. v $x=0$

3) b) $f(x) = \begin{cases} x^3 \sin^2\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$

a) spojitost: $\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} \underbrace{x^3}_{\to 0}\underbrace{\sin^2\left(\frac{1}{x}\right)}_{\text{omez.}} = 0$ (věta o ohraničených)

c) $f'(x) = 3x^2 \sin^2\frac{1}{x} + x^3\, 2\sin\frac{1}{x}\cdot\cos\frac{1}{x}\left(-\frac{1}{x^2}\right)$,

$\lim\limits_{x\to 0} f'(x) = 0$ (věta o oh.), f je spojitá v $a=0$ $\Rightarrow$

$\Rightarrow$ ex. $f'(0) = 0$

(nebo i $f'(0) = \lim\limits_{x\to 0} \frac{x^{\cancel{3}\,2}\sin^2\frac{1}{x}}{\cancel{x}\,1} = 0$

4) $\underline{f(x) = \begin{cases} e^{-\frac{1}{1-x^2}} & \text{pro } |x| < 1 \\ 0 & \text{pro } |x| \geq 1 \end{cases}}$ f je spojitá (spojitost elem. fce) v $(-1,1)$

— " — $(1,+\infty) \cup (-\infty,-1)$

spojitost v $x=1$ ($x=-1$ – fce je sudá)

$\lim\limits_{x\to 1-} f(x) = \lim\limits_{x\to 1-} e^{-\frac{1}{1-x^2}} \underset{VLSF}{=} \lim\limits_{t\to -\infty} e^t = 0$, $\cancel{\lim\limits_{x\to 1}} f(x) = f(0) \Rightarrow$ f je spojitá v $x = \pm 1$

$\lim\limits_{x\to 1-} -\frac{1}{1-x^2} = \text{"}\frac{-1}{0+}\text{"} = -\infty$

$f'(x) = 0 \quad v \ (1,+\infty) \cup (-\infty,-1)$

$$f'(x) = \left(e^{-\frac{1}{1-x^2}}\right)' = e^{-\frac{1}{1-x^2}} \cdot \left(\frac{1}{x^2-1}\right)' = e^{-\frac{1}{1-x^2}} \cdot \frac{-2x}{(x^2-1)}$$

$v \ (-1,1)$

$$\lim_{x\to 1-} e^{-\frac{1}{1-x^2}} \frac{-2x}{x^2-1} = \lim_{x\to 1} \underset{\to 2}{(2x)} \cdot \underbrace{e^{-\frac{1}{1-x^2}} \cdot \frac{1}{1-x^2}}_{\to 0} = 0, \quad \lim_{x\to 1+} f'(x) = 0$$

$$\left(= \lim_{x\to 1} f'(x)\right)$$

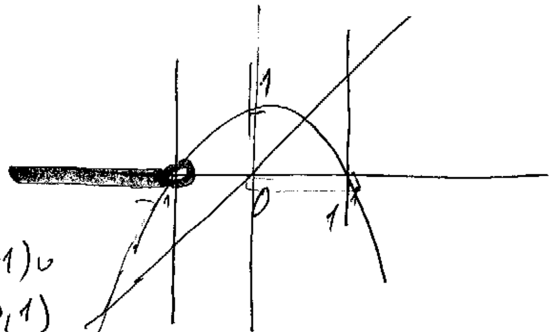
$$\left(\lim_{x\to 1-} e^{-\frac{1}{1-x^2}} \cdot \frac{1}{1-x^2} = \lim_{t\to\infty} e^{-t} \cdot t = \lim_{t\to\infty} \frac{t}{e^t} = 0 \right)$$

VLSF

$$\frac{1}{1-x^2} = t \to +\infty$$

$\Rightarrow$ i $f'(x)$ je spojitá v R

(5) $f(x) = \operatorname{arctg}^2\left(\sqrt{\frac{x}{1-x^2}}\right)$



$$D_f = \left\{x\in R;\ \frac{x}{1-x^2} > 0\right\} = (-\infty,-1) \cup (0,1)$$

- $x>0 \ \wedge \ 1-x^2>0 \Leftrightarrow x>0 \ \wedge \ x\in(-1,1) \ \ldots \ x\in(0,1)$

$\vee$ • $x<0 \ \wedge \ 1-x^2<0 \Leftrightarrow x\in(-\infty,-1)$

$$f'(x) = \cancel{2}\operatorname{arctg}\left(\sqrt{\frac{x}{1-x^2}}\right) \cdot \frac{1}{1+\frac{x}{1-x^2}} \cdot \frac{1}{\cancel{2}\sqrt{\frac{x}{1-x^2}}} \cdot \frac{1-x^2-x(-2x)}{(1-x^2)^2}$$

$$= \frac{2\operatorname{arctg}\sqrt{\frac{x}{1-x^2}}}{\sqrt{\frac{x}{1-x^2}}} \cdot \frac{1}{1+\frac{x}{1-x^2}} \cdot \frac{1+x^2}{(1-x^2)^2}, \quad x\in(-\infty,-1) \cup (0,+\infty)$$

$$\to 1 \qquad \to 1 \qquad \to 1$$

$$f'(0+) = \lim_{x\to 0+} f'(x) = \underline{1}$$

(f je spojitá v $0+$)

mlr z def. $\underline{f'(0+)} = \lim\limits_{x\to 0+} \dfrac{\operatorname{arctg}^2\left(\sqrt{\frac{x}{1-x^2}}\right)}{x} =$

$$= \lim_{x\to 0+} \underbrace{\frac{\operatorname{arctg}^2\left(\sqrt{\frac{x}{1-x^2}}\right)}{\left(\sqrt{\frac{x}{1-x^2}}\right)^2}}_{\to 1} \cdot \underbrace{\frac{\frac{\cancel{x}\,1}{1-x^2}}{\cancel{x}}}_{\to 1} = \underline{1}$$

6) viz v dnešním úkolu č 12

11) —"—

12) $\underline{\lim\limits_{x\to\infty} x^2 \arcsin\frac{1}{x}}$ „$\infty \cdot 0$" $= \lim\limits_{t\to 0+} \dfrac{\arcsin t}{t^2} = \lim\limits_{t\to 0+} \underbrace{\dfrac{\arcsin t}{t}}_{\to 1} \cdot \underbrace{\dfrac{1}{t}}_{\to +\infty} = \underline{+\infty}$

VLSF $\frac{1}{x} = t$

$\underline{\lim\limits_{x\to 0+} x^2 e^{\frac{1}{x}}}$ „$0+ \cdot +\infty$" $= \lim\limits_{x\to 0+} \dfrac{e^{\frac{1}{x}}}{\left(\frac{1}{x}\right)^2} \underset{\text{VLSF}}{=} \lim\limits_{t\to+\infty} \dfrac{e^t}{t^2} \underset{\text{l'H}\cdot 2\times}{=} \underline{+\infty}$

VLSF $\frac{1}{x} = t$

$\underline{\lim\limits_{x\to\infty} \left(\operatorname{arctg}\frac{1}{\sqrt{x}}\right)^{\frac{1}{\ln x}}}$ „0^0" $= \lim\limits_{x\to\infty} e^{\frac{1}{\ln x}\cdot \ln\left(\operatorname{arctg}\frac{1}{\sqrt{x}}\right)} \underset{\text{VLSF}}{=} \underline{e^{-\frac{1}{2}}}$

$\underline{\lim\limits_{x\to\infty} \dfrac{\ln\left(\operatorname{arctg}\frac{1}{\sqrt{x}}\right)}{\ln x}}$ „$\dfrac{-\infty}{\infty}$" $\underset{\text{l'H}}{=} \lim\limits_{x\to\infty} \dfrac{\frac{1}{\operatorname{arctg}\frac{1}{\sqrt{x}}} \cdot \frac{1}{1+\frac{1}{x}} \cdot \left(-\frac{1}{2}\frac{1}{\cancel{x}\sqrt{x}}\right)}{\frac{1}{\cancel{x}}}$

$$\underset{\substack{\text{VLSF}\\ \frac{1}{x}=t}}{=} \lim_{t\to 0+} \frac{1}{\operatorname{arctg}\sqrt{t}} \cdot \frac{1}{1+t} \cdot \left(-\frac{1}{2}\right)\sqrt{t} = \lim_{t\to 0+} \underbrace{\frac{\sqrt{t}}{\operatorname{arctg}\sqrt{t}}}_{\to 1} \left(-\frac{1}{2}\right) \underbrace{\frac{1}{1+t}}_{\to 1}$$

$$= \underline{\underline{-\frac{1}{2}}}$$

$$\lim_{x\to 0+} \left(\cos\sqrt{x}\right)^{\frac{2}{x}} = \lim_{x\to 0+} e^{\frac{2}{x}\ln(\cos\sqrt{x})} \underset{VLSF}{=} e^{-1}$$

$$\lim_{x\to 0+} \frac{2\ln(\cos\sqrt{x})}{x} = \lim_{x\to 0+} 2\cdot \underbrace{\frac{\ln(\cos\sqrt{x})}{\cos\sqrt{x}-1}}_{\to 1}\cdot \underbrace{\frac{\cos\sqrt{x}-1}{(\sqrt{x})^2}}_{\to -\frac{1}{2}} = -1$$

$$\lim_{x\to +\infty} \left(1-\frac{1}{x}\right)^{x} = \lim_{x\to\infty} e^{x\ln(1-\frac{1}{x})} = e^{-1}$$

$$\lim_{x\to +\infty} x\ln\left(1-\frac{1}{x}\right) \underset{\substack{VLSF\\ \frac{1}{x}=t}}{=} \lim_{t\to 0+} \frac{\ln(1-t)}{t} \underset{VLSF + \text{tabul}}{=} -1$$

$$\lim_{x\to 0} \underbrace{\left(e^{x}-2x\right)^{\frac{1}{2x}}}_{\text{"}1^{+\infty}\text{"}} = \lim_{x\to 0} e^{\frac{1}{2x}\ln(e^{x}-2x)} \underset{VLSF}{=} e^{-\frac{1}{2}}$$

$$\lim_{x\to 0} \frac{\ln(e^{x}-2x)}{2x} = \text{"}\frac{0}{0}\text{"} \underset{l'H}{\overset{*}{=}} \lim_{x\to 0} \frac{\overbrace{\frac{1}{e^{x}-2x}}^{\to 1}\overbrace{(e^{x}-2)}^{\to(-1)}}{2} = -\frac{1}{2}$$

nebo tabulkou: $\overset{*}{=} \lim_{x\to 0} \underbrace{\frac{\ln(e^{x}-2x)}{e^{x}-2x-1}}_{\substack{\to 1\\ \text{Tab. + VLSF}}}\cdot \frac{e^{x}-2x-1}{2x}$

$$\lim_{x\to 0} \frac{e^{x}-2x-1}{2x} = \lim_{x\to 0}\left(\frac{e^{x}-1}{2x} - 1\right) = \frac{1}{2} - 1 = -\frac{1}{2} \text{ cbd,}$$

Primeri!

(7) $f(x) = x e^{-|x-1|} = \begin{cases} x e^{-(x-1)} & \text{za } x \in (1,+\infty) \\ x e^{x-1} & \text{za } x \in (-\infty, 1\rangle \end{cases}$

ali $f(x) = x e^{-(x-1)\,\mathrm{sgn}(x-1)}$ za $x \neq 1$

$f(1) = 1$

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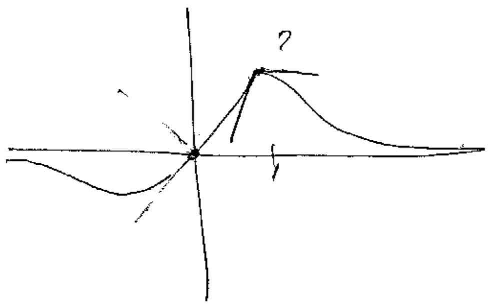
1) $D_f = \mathbb{R}$; $\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} \underbrace{x}_{+\infty} \underbrace{e^{-(x-1)}}_{\to 0} = \lim\limits_{x\to\infty} \dfrac{x}{e^{x-1}} = 0$

$\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} \underbrace{x}_{\to-\infty} e^{x-1} = \lim\limits_{x\to-\infty} \dfrac{x}{e^{1-x}} = 0$

oblika grafa: f je zvezna v $\mathbb{R}$, $f(x)=0 \Leftrightarrow x=0$

$f(x) > 0$ v $(0,+\infty)$, $f(x) < 0$ v $(-\infty, 0)$



2) $f'(x) = e^{-(x-1)} + x e^{-(x-1)} \cdot (-1) =$

$= e^{-(x-1)}(1-x)$, v $(1,+\infty)$; $\neq 0$ v $(1,+\infty)$

$f'(x) = (x e^{x-1})' = e^{x-1} + x e^{x-1} =$

$= e^{x-1}(1+x)$ v $(-\infty, 1)$

f ni odvedljiva v $x=1$: $f'(0+) = \lim\limits_{x\to 1+} f'(x) = 0$

$f'(0-) = \lim\limits_{x\to 1-} f'(x) = 1$

$f'(x) = 0 \Leftrightarrow x = -1$ v $(-\infty, 1)$

f'	$-$		$+$		$-$
	$\searrow$	-1	$\nearrow$	1	$\searrow$

v $x=-1$ je strogi lok. i glob. min

v $x=1$ je strogi lok. i glob. max

$f(1) = 1$ $f(-1) = -e^{-2}$

f

3) $f''(x) = e^{-(x-1)}(-1)(1-x) - e^{-(x-1)}$ u $(1,+\infty)$

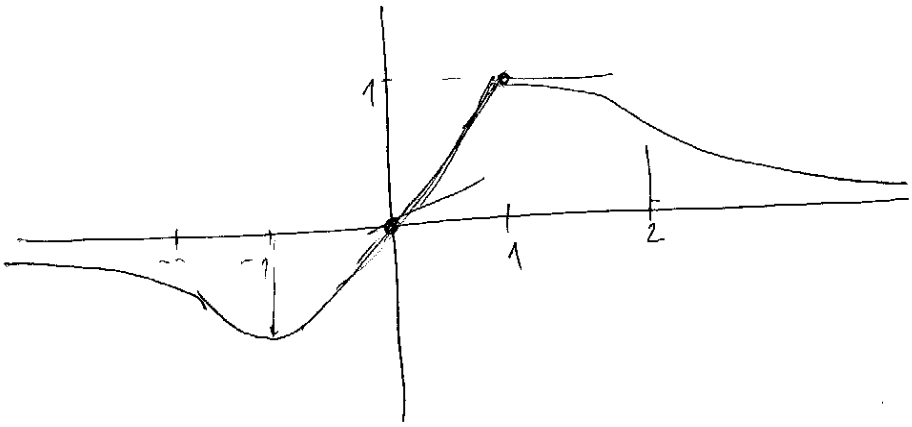
$= e^{-(x-1)}(x-2)$ $\quad f''(x)=0 \iff x=2$

$f''(x) = e^{x-1}(x+1) + e^{x-1} = e^{x-1}(x+2)$ u $(-\infty,+1)$ $\quad f''(x)=0 \iff x=-2$

		-2		1		2	
f''	$-$		$+$	o	$-$		$+$
	$\cap$		$\cup$		$\cap$		$-$

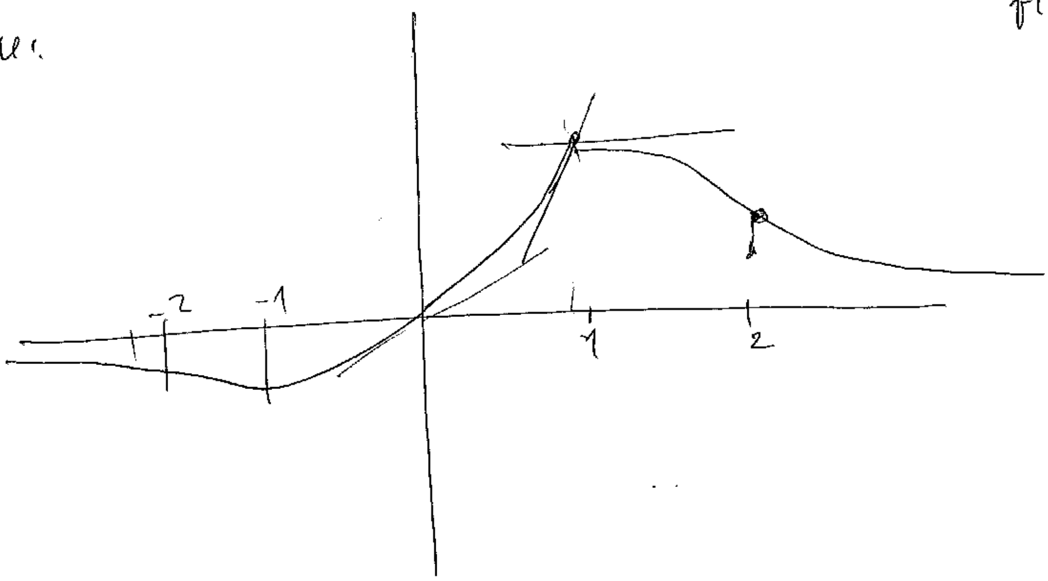
u $x=-2$ i u $x=2$ je infl. tač

$f'(0) = \frac{1}{e}$



$f(2) = 2e^{-1} = \frac{2}{e}$

lepe.



8. $f(x) = f(x) = \dfrac{|2x-1|}{(x-1)^2}$

1) $D_f = \mathbb{R} - \{1\} = (-\infty, 1) \cup (1, +\infty)$

$f(x) \geq 0$, $f(x) = 0 \iff x = \frac{1}{2}$ – glob. (i lok.) minimum ohe!

$\lim\limits_{x\to\pm\infty} \dfrac{|2x-1|}{(x-1)^2} = 0$, $\lim\limits_{x\to 1} \dfrac{|2x-1|}{(x-1)^2} = +\infty$ $\Rightarrow$ f nemá glob. max

2) $f'(x) = \left(\dfrac{2x-1}{(x-1)^2} \cdot \operatorname{sgn}(2x-1)\right)' \underset{x \neq \frac{1}{2}}{=} \operatorname{sgn}(2x-1) \dfrac{2(x-1)^{\cancel{2}} - (2x-1)\,2\cancel{(x-1)}}{(x-1)^{\cancel{4}\,3}}$

$= \operatorname{sgn}(2x-1) \dfrac{2(-x)}{(x-1)^3} = \operatorname{sgn}(2x-1) \dfrac{-2x}{(x-1)^3}$

$f'(\frac{1}{2}\pm) = \lim\limits_{x\to\frac{1}{2}\pm} f'(x) = \pm \dfrac{-\frac{1}{2}\cdot 2}{-\frac{1}{8}} = \pm 4 \cdot 2 = \underline{\pm 8}$ $\Rightarrow$ derivace v $x = \frac{1}{2}$ fce nemá!

3) $f'(x) = 0 \iff x = 0$

$$\begin{array}{ccccccccc} & + & & - & & + & & - & \\ \hline & \nearrow & 0 & \searrow & \frac{1}{2} & \nearrow & 1 & \searrow & \end{array}$$

v $x = 0$ je ostré lok. max

4) $f''(x) = \operatorname{sgn}(2x-1)(-2) \dfrac{(x-1)^{\cancel{3}} - x\, 3\cancel{(x-1)^2}}{(x-1)^{\cancel{6}\,4}} = +2 \operatorname{sgn}(2x-1) \dfrac{2x+1}{(x-1)^4}$

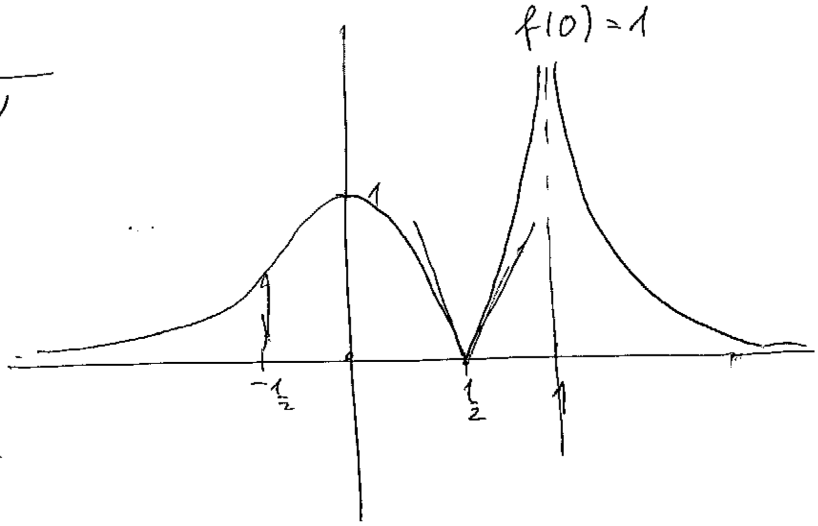
$x \neq \frac{1}{2}$

$f''(x) = 0 \iff x = -\frac{1}{2}$

$$\begin{array}{cccccccccc} f'' & + & & - & & + & & + & \\ \hline & \cup & -\frac{1}{2} & \cap & \frac{1}{2} & \cup & 1 & \cup & \end{array}$$

v $x = -\frac{1}{2}$ je infl. bod

$f(0) = 1$



(9) $f(x) = \exp\left(\frac{x^2+1}{x^2-1}\right)$

a) $D_f = (-\infty,-1)\cup(-1,1)\cup(1,+\infty)$, fce spojitá, sudá, kladná

$\lim_{x\to\pm\infty} \exp\left(\frac{x^2+1}{x^2-1}\right) = e$, $\lim_{x\to 1+} e^{\frac{x^2+1}{x^2-1}} = \lim_{y\to+\infty} e^y = +\infty \Rightarrow$ f nemá glob. max

$f(0) = \frac{1}{e}$ $\lim_{x\to 1-} e^{\frac{x^2+1}{x^2-1}} = \lim_{y\to-\infty} e^y = 0 \Rightarrow$ f nemá glob. min.

b) $f'(x) = e^{\frac{x^2+1}{x^2-1}} \cdot \frac{2x(x^2-1)-(x^2+1).2x}{(x^2-1)^2} = \frac{-4x}{(x^2-1)^2}\, e^{\frac{x^2+1}{x^2-1}}$

$f'(x)=0 \Leftrightarrow x=0$, f': $+$ $\circ$ $+$ $|$ $-$ $\circ$ $-$; $\nearrow$ -1 $\nearrow$ 0 $\searrow$ 1 $\searrow$ $\Rightarrow$ v $x=0$ je ostré lok. i ~~g~~ maximum

c) ~~$f''(x) = e^{\frac{x^2+1}{x^2-1}}\left[\left(\frac{-4x}{(x^2-1)^2}\right)^2 - 4\,\frac{(x^2-1)^2 - x\,2(x^2-1)\cdot 2x}{(x^2-1)^3}\right] =$~~

~~$= \frac{e^{\frac{x^2+1}{x^2-1}}}{(x^2-1)^4}\left(16x^2 - 4\left((x-1)^2 - 4x^2(x^2-1)\right)\right)$~~

~~$= \frac{e^{\frac{x^2+1}{x^2-1}}}{(x^2-1)^4}\left(16x^2 - 4(x^2-2x+1-4x^4+4x^2)\right)$~~

~~$= -\,\text{''}\,-\,(16x^2 - 4x^2 + 8x - 4 + 16x^4 - 16x^2$~~

$f''(x) = e^{\frac{x^2+1}{x^2-1}}\left[\left(\frac{-4x}{(x^2-1)^2}\right)^2 - 4\,\frac{(x^2-1)^2 - x\,2(x^2-1).2x}{(x^2-1)^4}\right] =$

$= \frac{e^{\frac{x^2+1}{x^2-1}}}{(x^2-1)^4}\left[16x^2 - 4\left(x^4-2x^2+1-4(x^4-x^2)\right)\right]$

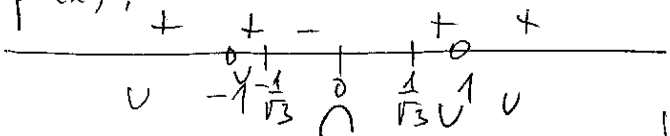
$16x^2 - 4(-3x^4+2x^2+1)] =$

$= 12x^4 + 8x^2 - 4 = 4(3x^2+2x^2-1)$

$x^2_{1/2} = \frac{-2\pm\sqrt{16}}{6} = \frac{2}{6} = \frac{1}{3}$

$x = \pm\frac{\sqrt{3}}{3}$

$f''(x)$:



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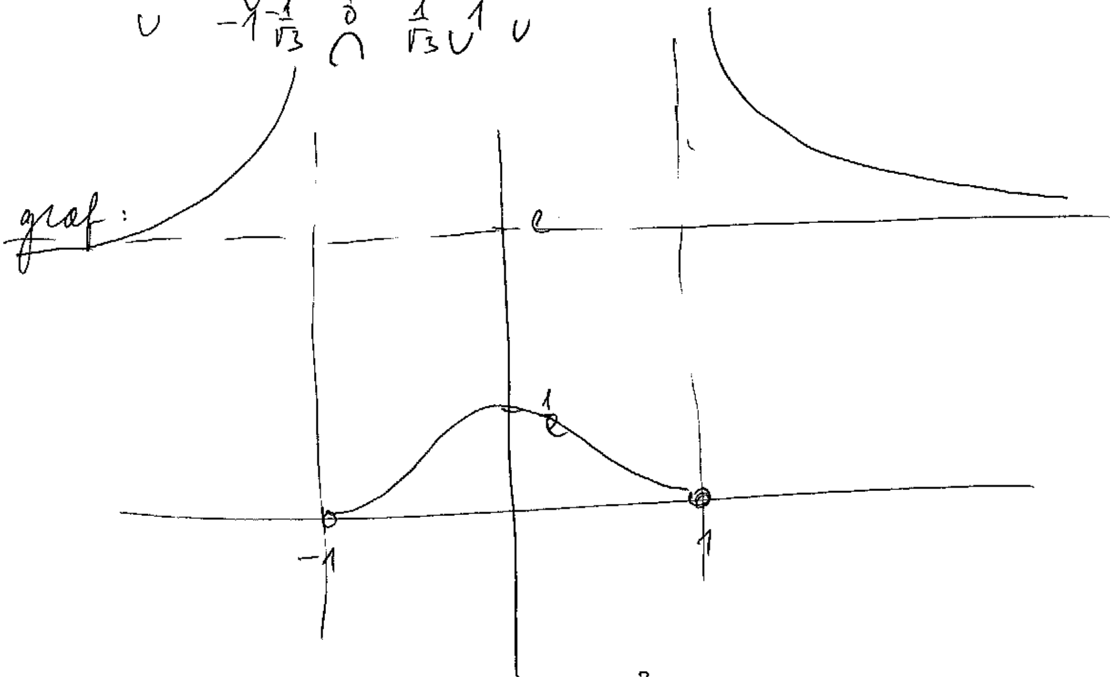


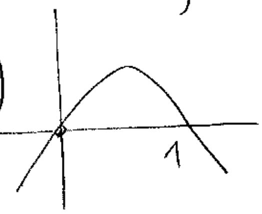
$$\lim_{x\to 1-} f'(x) = \lim_{x\to 1-} \frac{-4x}{(x^2-1)^2}\, e^{\frac{x^2+1}{x^2-1}} = \ell\ (-4x)\,\frac{1}{(x^2-1)^2}\left(e^{\frac{x^2+1}{x^2-1}}\right) = 0\ ?$$

(10) $f(x) = \ln(|x| - x^2)$ – fce sudá, stačí v $(0,1)$

a) $D_f = \{x;\ |x| - x^2 > 0\} = (-1,0)\cup(0,1)$

stačí pro $x>0$ $\quad x - x^2 > 0 \Leftrightarrow x\in(1,0)$

$x(x-1) > 0$



spojitá v D_f, $\quad |x| < x^2 < 1$ $\quad$ ~~(max v $x=\frac12$ $f(\frac12) = \ln(\frac14) < 0$~~

max pro $x = \frac12$ $\quad \frac12 - \frac14 \le \frac14 < 1$

$\Rightarrow f(x) < 0$ v D_f

$$\left.\begin{aligned}\lim_{x\to 0+} \ln(|x| - x^2) &= \lim_{t\to 0+} \ln t = -\infty\\ \lim_{x\to 1-} \ln(x - x^2) &= \ell \cdot \lim_{t\to 0+}\ln t = -\infty\end{aligned}\right\} \Rightarrow f \text{ nemá glob. min}$$

b) $f'(x) = \frac{1}{x-x^2}(1-2x)$
$x \in (0,1)$

$f'(x) = 0 \iff x = \frac{1}{2}$, $f'(x) > 0$ v $(0, \frac{1}{2})$ $f \nearrow$ v $(0, \frac{1}{2})$
$f'(x) < 0$ v $(\frac{1}{2}, 1)$ $f \searrow$ v $(\frac{1}{2}, +\infty)$

$\Rightarrow$ v $x = \frac{1}{2}$ ostré lok. i glob. maxi (neostré), (v $x = -\frac{1}{2}$ tiež)

$f(\pm \frac{1}{2}) = \ln(\frac{1}{4})$

c) $f''(x) = \frac{-2(x-x^2) - (1-2x)(1-2x)}{(x-x^2)^2} = \frac{-2x + 2x^2 - (4x^2 - 4x + 1)}{(x-x^2)^2} =$

$= \frac{-2x^2 + 2x - 1}{(x-x^2)^2} = -\frac{2x^2 - 2x + 1}{(x-x^2)^2}$, $D = 4 - 8 < 0$ nemá nulové body

$f''(x) < 0$ v $(-1, 0)$ i v $(0,1)$ $\Rightarrow$ $f \cap$ v $(-1,0)$ i v $(0,1)$

graf: